Technology Company Enterprise Value
Technology Companies - Part IV

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In this white paper we will employ the equations from the previous white papers on technology companies (Modeling Technology Product Revenue, Capitalizing Research And Development Expenditures, and Technology Company ROI And NPV) to calculate technology company enterprise value. To that end we will work through the following hypothetical problem...

Our Hypothetical Problem

The table below presents our hypothetical company's balance sheet and income statement at and for the twelve months ended December 31, 2023 (Dollars are in thousands)...

Balance Sheet

Description	Balance
Cash and securities	37,500
Working capital	200,000
Fixed assets (net)	500,000
Intangible assets	150,000
Other assets	100,000
Total assets	**987,500**
NIB liabilities	112,500
IB debt	200,000
Equity capital	675,000
Total financing	**987,500**

Income Statement

Description	Balance
Operating revenue	1,000,000
Operating expense	-550,000
R&D expense	-180,000
Non-operating income	5,000
Interest expense	-15,000
Pre-tax income	**260,000**
Income tax expense	-45,500
Net income	**214,500**

Rates and Ratios

Description	Balance
Capital assets ($)	800,000
Annualized revenue ($)	1,000,000
Net operating income ($)	450,000
Ratio cap assets to revenue	0.8000
Pre-tax revenue margin	0.4500
R&D expense to revenue	0.1800
Income tax rate	0.1750

Assume that weighted-average revenue life is 3 years, the after-tax cost of capital is 12.50%, and the secular go-forward revenue growth rate is 5.00%.

Questions:
Question 1: What is enterprise value via a ROI-based valuation model?
Question 2: What is enterprise value via a NPV-based valuation model?
Question 3: What is the major potential weakness of this analysis?

Notes: Capital assets are defined as total assets minus cash and securities, intangible assets and non-operating assets. Pre-tax net operating income excludes research and development expenditures and non-operating items. Revene margin is defined as the ratio of operating revenue minus operating expense to operating revenue.

Product Revenue

We will define operating revenue to be GAAP revenue minus non-operating items (interest income, investment gain/(loss), etc.). We will define the variable R_s^s to be annualized operating revenue at time s on a product brought to market at time s. The equation for product annualized revenue is... [1]

$$R_s^s = \text{Periodic operating revenue} \times \Delta^{-1} \text{ ...where... } \Delta = \text{Period length in years} \tag{1}$$

We will define the variable L_s^s to be lifetime revenue at time s on a product brought to market at time s, the variable λ to be the rate to technological obsolesence, and the variable β to be the product's weighted-average revenue life in years. Using Equation (1) above, the equation for product lifetime revenue at time s is... [1]

$$L_s^s = R_s^s \, \lambda^{-1} \;\; \text{...where...} \;\; \lambda = \frac{1}{\beta} \tag{2}$$

Using Equations (1) and (2) above, the equations for product annualized and lifetime revenue at time t on a product brought to market at time s are... [1]

$$R_t^s = R_s^s \, \mathrm{Exp}\left\{ -\lambda \, (t-s) \right\} \;\; \text{...such that...} \;\; L_t^s = R_t^s \, \lambda^{-1} = L_s^s \, \mathrm{Exp}\left\{ -\lambda \, (t-s) \right\} \tag{3}$$

Note that the equation for the derivatives of Equation (3) above with respect to time t are... [1]

$$\frac{\delta}{\delta t} \, R_t^s = -\lambda \, R_s^s \, \mathrm{Exp}\left\{ -\lambda \, (t-s) \right\} \;\; \text{...and...} \;\; \frac{\delta}{\delta t} \, L_t^s = -\lambda \, L_s^s \, \mathrm{Exp}\left\{ -\lambda \, (t-s) \right\} \tag{4}$$

We will define the variable $R_{m,n}^s$ to be cumulative revenue recognized over the time interval $[m, n]$ on a product brought to market at time s. Using Equation (4) above, the equation for cumulative revenue is... [1]

$$R_{m,n}^s = \int\limits_m^n \frac{\delta}{\delta t} \, L_t^s \, \delta t = R_s^s \left(\mathrm{Exp}\left\{ -\lambda \, (m-s) \right\} - \mathrm{Exp}\left\{ -\lambda \, (n-s) \right\} \right) \lambda^{-1} \tag{5}$$

Using Equation (2) above, we can rewrite Equation (5) above as...

$$R_{m,n}^s = L_s^s \left(\mathrm{Exp}\left\{ -\lambda \, (m-s) \right\} - \mathrm{Exp}\left\{ -\lambda \, (n-s) \right\} \right) \tag{6}$$

Product On-Balance Sheet Investment

We will define capital assets to be total GAAP assets minus cash, marketable securities, intangible assets, and non-operating assets, and the variable ϕ to be the ratio of capital assets to annualized revenue. The equation for the variable ϕ is... [3]

$$\phi = \frac{\text{Capital assets}}{\text{Annualized revenue}} \;\; \text{...where...} \;\; \text{Annualized revenue} = \text{Periodic revenue} \times \Delta^{-1} \tag{7}$$

We will define the variable A_t^s to be capital assets at time t on a product brought to market at time s. Using Equations (2), (3) and (7) above, the equation for capital assets is... [3]

$$A_t^s = \phi \, R_t^s = \phi \, \lambda \, L_t^s = \phi \, \lambda \, L_s^s \, \mathrm{Exp}\left\{ -\lambda \, (t-s) \right\} \tag{8}$$

As product revenue decreases over time, capital assets also decrease, which is a source of cash flow. Using Equations (4) and (8) above, the equation for the change in capital assets over time is... [3]

$$\frac{\delta}{\delta t} \, A_t^s = \phi \, \frac{\delta}{\delta t} \, R_t^s = -\lambda \, \phi \, \lambda \, L_s^s \, \mathrm{Exp}\left\{ -\lambda \, (t-s) \right\} \;\; \text{...such that...} \;\; \delta A_t^s = -\lambda \, \phi \, \lambda \, L_s^s \, \mathrm{Exp}\left\{ -\lambda \, (t-s) \right\} \delta t \tag{9}$$

We will define the variable $A_{m,n}^s$ to be cumulative return of capital investment over the time interval $[m, n]$ on a product brought to market at time s. Using Equation (9) above, the equation for the return of capital investment over the time interval $[m, n]$ is... [3]

$$A_{m,n}^s = -\int\limits_m^n \frac{\delta}{\delta t} \, A_t^s \, \delta t = \phi \, R_s^s \left(\mathrm{Exp}\left\{ -\lambda \, (m-s) \right\} - \mathrm{Exp}\left\{ -\lambda \, (n-s) \right\} \right) \tag{10}$$

Using Equation (2) above, we can rewrite Equation (10) above as...

$$A_{m,n}^s = \phi \, \lambda \, L_s^s \left(\mathrm{Exp}\left\{ -\lambda \, (n-s) \right\} - \mathrm{Exp}\left\{ -\lambda \, (m-s) \right\} \right) \tag{11}$$

Product Off-Balance Sheet Investment

We will define the variable ω to be the dollar investment in pre-tax research and development today to product one dollar of revenue in the future. The equation for the variable ω is... [2]

$$\omega = \frac{\text{Periodic research and development expense}}{\text{Periodic revenue}} \tag{12}$$

We will define the variable D_t^s to be capitalized after-tax research and development expenditures at time t on a product brought to market at time s, and the variable α to be the income tax rate. Using Equations (2), (3) and (12) above, the equation for capitalized research and development is... [2]

$$D_t^s = \omega\,(1-\alpha)\,L_t^s = \omega\,(1-\alpha)\,L_s^s\,\mathrm{Exp}\left\{-\lambda\,(t-s)\right\} \tag{13}$$

As product revenue decreases over time, capitalized research and development also decreases via amortization expense, which is a non-cash item. Using Equations (4) and (13) above, the equation for the change in capitalized research and develpment over time is... [2]

$$\frac{\delta}{\delta t}\,D_t^s = \omega\,(1-\alpha)\,\frac{\delta}{\delta t}\,L_t^s = -\lambda\,\omega\,(1-\alpha)\,L_s^s\,\mathrm{Exp}\left\{-\lambda\,(t-s)\right\} \text{ ...such that... } \delta D_t^s = -\lambda\,\omega\,(1-\alpha)\,L_s^s\,\mathrm{Exp}\left\{-\lambda\,(t-s)\right\}\delta t \tag{14}$$

We will define the variable $D_{m,n}^s$ to be the cumulative change in off-balance sheet investment (i.e. amortization of after-tax capitalized R&D) over the time interval $[m,n]$ on a product brought to market at time s. Using Equation (14) above, the equation for the cumulative change in off-balance sheet investment, which in this case is a non-cash expense, over the time interval $[m,n]$ is... [2]

$$D_{m,n}^s = -\int\limits_m^n \frac{\delta}{\delta t}\,D_t^s\,\delta t = \omega\,(1-\alpha)\,R_s^s\left(\mathrm{Exp}\left\{-\lambda\,(m-s)\right\} - \mathrm{Exp}\left\{-\lambda\,(n-s)\right\}\right)\lambda^{-1} \tag{15}$$

Using Equation (2) above, we can rewrite Equation (15) above as...

$$D_{m,n}^s = \omega\,(1-\alpha)\,L_s^s\left(\mathrm{Exp}\left\{-\lambda\,(m-s)\right\} - \mathrm{Exp}\left\{-\lambda\,(n-s)\right\}\right) \tag{16}$$

Product Net Income, Cash Flow, NPV and ROI

We will define the variable M_t^s to be annualized after-tax operating revenue margin at time t on a product brought to market at time s, and the variable θ to be revenue margin, which is defined as the ratio of operating revenue minus operating expense to operating revenue. Using Equations (1) and (2) above, the equation for annualized after-tax revenue margin is... [3]

$$M_t^s = \theta\,(1-\alpha)\,R_t^s = \lambda\,\theta\,(1-\alpha)\,L_t^s \tag{17}$$

We will define the variable $N_{m,n}^s$ to be cumulative net income over the time interval $[m,n]$ on a product brought to market at time s. Net income is defined as after-tax revenue margin minus amortization of capitalized after-tax research and development expenditures. Using Equations (14) and (17) above, the equation for cumulative net income is... [3]

$$N_{m,n}^s = \int\limits_m^n M_t^s\,\delta t - \int\limits_m^n \delta D_t^s = (\omega-\theta)\,(1-\alpha)\,\lambda^{-1}R_s^s\left(\mathrm{Exp}\left\{-\lambda\,(n-s)\right\} - \mathrm{Exp}\left\{-\lambda\,(m-s)\right\}\right) \tag{18}$$

Using Equation (2) above, we can rewrite Equation (18) above as...

$$N_{m,n}^s = (\omega-\theta)\,(1-\alpha)\,L_s^s\left(\mathrm{Exp}\left\{-\lambda\,(n-s)\right\} - \mathrm{Exp}\left\{-\lambda\,(m-s)\right\}\right) \tag{19}$$

We will define the variable $C_{m,n}^s$ to be cumulative after-tax cash flow over the time interval $[m,n]$ on a product brought to market at time s. Using Equations (8) and (17) above, the equation for cash flow is... [3]

$$C_{m,n}^s = \int\limits_m^n M_t^s\,\delta t - \int\limits_m^n \delta A_t^s = (\theta\,(1-\alpha)\,\lambda^{-1} + \phi)\,R_s^s\left(\mathrm{Exp}\left\{-\lambda\,(m-s)\right\} - \mathrm{Exp}\left\{-\lambda\,(n-s)\right\}\right) \tag{20}$$

Using Equation (2) above, we can rewrite Equation (20) above as...

$$C^s_{m,n} = (\theta\,(1-\alpha) + \phi\,\lambda)\,L^s_s\Bigg(\text{Exp}\Bigg\{-\lambda\,(m-s)\Bigg\} - \text{Exp}\Bigg\{-\lambda\,(n-s)\Bigg\}\Bigg) \tag{21}$$

We will define the function $NPV(s)$ to be product net present value at time s of product net cash flow over the time interval $[s, \infty]$ on a product brought to market at time s, and the variable κ to be the risk-adjusted discount rate. Using Equations (8), (13) and (20) above, the equation for product net present value is... [3]

$$NPV(s) = (\theta\,(1-\alpha) + \lambda\,\phi)\,R^s_s \int\limits_s^\infty \text{Exp}\Bigg\{-(\lambda+\kappa)\,(t-s)\Bigg\}\,\delta t - \Bigg(A^s_s + D^s_s\Bigg) \tag{22}$$

The solution to Equation (22) above is... [3]

$$NPV(s) = \Bigg(\lambda\,(\lambda+\kappa)^{-1}(\theta\,(1-\alpha)+\phi\,\lambda) - (\lambda\,\phi + \omega\,(1-\alpha))\Bigg)L^s_s \tag{23}$$

We will define the function ROI to be product after-tax, continuous-time return on investment. The equation for ROI is... [3]

$$ROI = (\theta\,(1-\alpha) + \phi\,\lambda)\,/\,(\phi + \omega\,(1-\alpha)\,\lambda^{-1}) - \lambda \tag{24}$$

Company Revenue

We will define the variable R_t to be company annualized revenue at time t and the variable μ to be the continuous-time revenue growth rate. The equation for company annualized revenue at time t is...

$$R_t = R_0\,\text{Exp}\Bigg\{\mu\,t\Bigg\}\ \text{...such that...}\ \delta R_t = \mu\,R_0\,\text{Exp}\Bigg\{\mu\,t\Bigg\}\delta t \tag{25}$$

We will define the variable L_t to be company lifetime revenue at time t. Using Equations (2) and (25) above, the equation for company lifetime revenue at time t is...

$$L_t = R_0\,\text{Exp}\Bigg\{\mu\,t\Bigg\}\lambda^{-1}\ \text{...such that...}\ \delta L_t = \mu\,R_0\,\text{Exp}\Bigg\{\mu\,t\Bigg\}\lambda^{-1}\delta t \tag{26}$$

We will define the variable X_t to be lifetime revenue on new products brought to market at time t. Using Equation (26) above, the equation for the change in company lifetime revenue over the time interval $[t, t+\delta t]$ is...

$$\delta L_t = X_t\,\delta t - R_t\,\delta t \tag{27}$$

If we equate change in lifetime revenue Equations (26) and (27) above, then using Appendix Equation (49) below, the equation for lifetime revenue on new products brought to market at time t is...

$$X_t = R_0\,\text{Exp}\Bigg\{\mu\,t\Bigg\}\Bigg(1+\mu\,\lambda^{-1}\Bigg) \tag{28}$$

Using Equations (25) and (28) above, we can rewrite Equation (27) above as...

$$\delta L_t = R_0\,\text{Exp}\Bigg\{\mu\,t\Bigg\}\Bigg(1+\mu\,\lambda^{-1}\Bigg) - \mu\,R_0\,\text{Exp}\Bigg\{\mu\,t\Bigg\}\delta t \tag{29}$$

Company Investment

We will define the variable I_t to be total on-balance sheet and off-balance sheet investment at time t. Using Equations (8), (13), (25) and (26) above, the equation for company total investment at time t is...

$$I_t = \phi\,R_t + \omega\,(1-\alpha)\,R_t\,\lambda^{-1} = L_t\Bigg(\lambda\,\phi + \omega\,(1-\alpha)\Bigg) \tag{30}$$

Using Equations (26) and (30) above, the equation for the change in company investment over the time interval $[t, t+\delta t]$ is...

$$\delta I_t = \frac{\delta}{\delta t}\, L_t \bigg(\lambda\,\phi + \omega\,(1-\alpha)\bigg)\delta t = \mu\, R_0\, \mathrm{Exp}\bigg\{\mu\, t\bigg\}\bigg(\lambda\,\phi + \omega\,(1-\alpha)\bigg)\lambda^{-1}\delta t \tag{31}$$

We will define the variable Y_t to be the investment required to bring new products to market at time t. Using Equation (31) above, the equation for the change in company investment over the time interval $[t, t+\delta t]$ is...

$$\delta I_t = Y_t\,\delta t - \phi\, R_t\,\delta t \tag{32}$$

If we equate change in investment Equations (31) and (32) above, then using Appendix Equation (50) below, the equation for investment on new products brought to market at time t is...

$$Y_t = R_0\, \mathrm{Exp}\bigg\{\mu\, t\bigg\}\bigg[\mu\,\lambda^{-1}\bigg(\lambda\,\phi + \omega\,(1-\alpha)\bigg) + \phi\bigg] \tag{33}$$

Using Equations (25) and (33) above, we can rewrite Equation (32) above as...

$$\delta I_t = R_0\, \mathrm{Exp}\bigg\{\mu\, t\bigg\}\bigg[\mu\,\lambda^{-1}\bigg(\lambda\,\phi + \omega\,(1-\alpha)\bigg) + \phi\bigg] - \phi\, R_0\, \mathrm{Exp}\bigg\{\mu\, t\bigg\} \tag{34}$$

ROI Valuation Model

We will define the variable Γ to be the ratio of total investment to annualized revenue. Using Equation (**??**) above, the equation for this ratio is...

$$\Gamma = I_t \Big/ R_t = \phi\, R_t + \omega\,(1-\alpha)\, R_t\,\lambda^{-1} \Big/ R_t = \phi + \omega\,(1-\alpha)\,\lambda^{-1} \tag{35}$$

We will define the variable V_0 to be company enterprise value at time zero. If we recast the dividend discount model as a return model then we get the following valuation equation... [4]

$$V_0 = \Gamma R_0\,\frac{\pi - \mu}{\kappa - \mu} \tag{36}$$

Our ROI valuation model parameters are...

Symbol	Description	Notes
R_0	Annualized revenue at time zero	Annualized company operating revenue
π	Continuous-time return on investment	After-tax ROI
μ	Continous-time investment growth rate	Assume that investment grows at same rate as revenue
κ	Continous-time cost of capital	Discount rate applied to after-tax cash flow
Γ	Ratio of investment to annualized revenue	Equation (35) above

NPV Valuation Model

We will define the variable $V(1)$ to be the net present value of lifetime revenue at time zero. Using Equations (23) and (26) above, the equation for the npv of lifetime revenue at time zero is...

$$V(1) = \bigg(\lambda\,(\lambda+\kappa)^{-1}(\theta\,(1-\alpha) + \phi\,\lambda)\bigg)L_0 \tag{37}$$

Note that in Equation (37) above we eliminated the investment portion of the npv equation because that investment has already been made as of time zero (on-balance sheet and off-balance sheet investment).

We will define the variable $V(2)$ to be the net present value of lifetime revenue applicable to new products brought to market over the time interval $[0, \infty]$. Using Equations (23) and (28) above, the equation for the npv of future lifetime revenue is...

$$V(2) = \int\limits_0^\infty \bigg(\lambda\,(\lambda+\kappa)^{-1}(\theta\,(1-\alpha) + \phi\,\lambda) - (\lambda\,\phi + \omega\,(1-\alpha))\bigg)X_t\, \mathrm{Exp}\bigg\{-\kappa\, t\bigg\}\,\delta t \tag{38}$$

Using Appendix Equation (52) below, the solution to Equation (38) above is...

$$V(2) = R_0 \bigg(1 + \mu\,\lambda^{-1}\bigg) \bigg(\lambda\,(\lambda + \kappa)^{-1}(\theta\,(1 - \alpha) + \phi\,\lambda) - (\lambda\,\phi + \omega\,(1 - \alpha))\bigg) \bigg(\kappa - \mu\bigg)^{-1} \tag{39}$$

Using Equations (37) and (39) above, the equation for enterprise value at time zero is...

$$V_0 = V(1) + V(2) \tag{40}$$

Answers To Our Hypothetical Problem

Our model parameters are...

Symbol	Description	Value	Notes
R_0	Annualized revenue at time zero	1,000,000	See [Rates and Ratios] above
α	Income tax rate	0.1750	See [Rates and Ratios] above
β	Weighted-average product revenue life in years	3.0000	Base assumption
κ	Continuous-time cost of capital	0.1178	$\ln(1 + 0.1250)$
λ	Rate of technological obsolsence	0.3333	Equation (2) above
μ	Continous-time revenue growth rate	0.0488	$\ln(1 + 0.0500)$
ω	R&D expense to revenue	0.1800	See [Rates and Ratios] above
ϕ	Ratio of on-balance sheet assets to revenue	0.8000	See [Rates and Ratios] above
θ	Revenue margin	0.4500	See [Rates and Ratios] above

Using Equation (24) above and the model parameters in the table above, the equation for product ROI is...

$$\pi = (0.4500 \times (1 - 0.1750) + 0.8000 \times 0.3333)\,/\,(0.8000 + 0.1800 \times (1 - 0.1750) \times 0.3333^{-1}) - 0.3333 = 0.1788 \tag{41}$$

Using Equation (**??**) and the model parameters in the table above, the equation for lifetime revenue at time zero is...

$$L_0 = 1,000,000 \times 0.3333^{-1} = 3,000,000 \tag{42}$$

Using Equation (35) and the model parameters in the table above, the equation for the ratio of total investment to annualized revenue is...

$$\Gamma = 0.8000 + 0.1800 \times (1 - 0.1750) \times 0.3333^{-1} = 1.2455 \tag{43}$$

Question 1: What is enterprise value via a ROI-based valuation model?

Using Equation (36), (41) and (43) above and the model parameters in the table above, the answer to the question is...

$$V_0 = 1.2455 \times 1,000,000 \times \frac{0.1788 - 0.0488}{0.1178 - 0.0488} = 2,346,600 \tag{44}$$

Question 2: What is enterprise value via a NPV-based valuation model?

Using Equations (37) and (42) above and the model parameters in the table above, the equation for the npv of lifetime revenue at time zero is...

$$\begin{aligned} V(1) &= \bigg(0.3333 \times (0.3333 + 0.1178)^{-1} \times (0.4500 \times (1 - 0.1750) + 0.8000 \times 0.3333)\bigg) \times 3,000,000 \\ &= 1,414,085 \end{aligned} \tag{45}$$

Using Equation (39) and the model parameters in the table above, the equation for the npv of future lifetime revenue is...

$$\begin{aligned} V(2) = 1,000,000 \times \bigg(1 + 0.0488 \times 0.3333^{-1}\bigg) \times \bigg(&0.3333 \times (0.3333 + 0.1178)^{-1} \times (0.4500 \times (1 - 0.1750) \\ &+ 0.8000 \times 0.3333) - (0.3333 \times 0.8000 + 0.1800 \times (1 - 0.1750))\bigg) \times \bigg(0.1178 - 0.0488\bigg)^{-1} \\ &= 933,721 \end{aligned} \tag{46}$$

Using Equations (40), (45) and (46) above, the answer to the question is...

$$V_0 = 1,414,085 + 933,721 = 2,346,600 \tag{47}$$

Question 3: What is the major potential weakness of this analysis?

In the long-term, economic profits (excess of return on investment over the cost of capital) go to zero. The above analysis assumes that a return on investment of 0.1788, which is significantly greater than the cost of capital of 0.1178, can be maintained forever, which in a competitive market is impossible.

Appendix

A. The solution to the following integral is...

$$\int\limits_0^\infty \mathrm{Exp}\left\{(\mu-\kappa)\,t\right\}\delta t = \frac{1}{\mu-\kappa}\left(\mathrm{Exp}\left\{(\mu-\kappa)\times\infty\right\} - \mathrm{Exp}\left\{(\mu-\kappa)\times 0\right\} = \left(\kappa-\mu\right)^{-1} \tag{48}$$

B. Using Equations (26) and (27) above, the solution to the following equation is...

$$\begin{aligned}\delta L_t &= X_t\,\delta t - R_t\,\delta t\\ \mu\,R_0\,\mathrm{Exp}\left\{\mu\,t\right\}\lambda^{-1}\,\delta t &= X_t\,\delta t - R_0\,\mathrm{Exp}\left\{\mu\,t\right\}\delta t\\ \mu\,R_0\,\mathrm{Exp}\left\{\mu\,t\right\}\lambda^{-1} + R_0\,\mathrm{Exp}\left\{\mu\,t\right\} &= X_t\\ R_0\,\mathrm{Exp}\left\{\mu\,t\right\}\left(1+\mu\,\lambda^{-1}\right) &= X_t\end{aligned} \tag{49}$$

C. Using Equations (31) and (32) above, the solution to the following equation is...

$$\begin{aligned}\delta I_t &= Y_t\,\delta t - \phi\,R_t\,\delta t\\ \mu\,R_0\,\mathrm{Exp}\left\{\mu\,t\right\}\left(\lambda\,\phi+\omega\,(1-\alpha)\right)\lambda^{-1}\delta t &= Y_t\,\delta t - \phi\,R_0\,\mathrm{Exp}\left\{\mu\,t\right\}\delta t\\ \mu\,R_0\,\mathrm{Exp}\left\{\mu\,t\right\}\left(\lambda\,\phi+\omega\,(1-\alpha)\right)\lambda^{-1} + \phi\,R_0\,\mathrm{Exp}\left\{\mu\,t\right\} &= Y_t\\ R_0\,\mathrm{Exp}\left\{\mu\,t\right\}\left[\mu\,\lambda^{-1}\left(\lambda\,\phi+\omega\,(1-\alpha)\right)+\phi\right] &= Y_t\end{aligned} \tag{50}$$

D. The solution to the following integral is...

$$\begin{aligned}I &= \int\limits_0^\infty \Big(\lambda\,(\lambda+\kappa)^{-1}(\theta\,(1-\alpha)+\phi\,\lambda) - (\lambda\,\phi+\omega\,(1-\alpha))\Big)X_t\,\mathrm{Exp}\left\{-\kappa\,t\right\}\delta t\\ &= \int\limits_0^\infty \Big(\lambda\,(\lambda+\kappa)^{-1}(\theta\,(1-\alpha)+\phi\,\lambda) - (\lambda\,\phi+\omega\,(1-\alpha))\Big)R_0\,\mathrm{Exp}\left\{\mu\,t\right\}\left(1+\mu\,\lambda^{-1}\right)\mathrm{Exp}\left\{-\kappa\,t\right\}\delta t\\ &= R_0\left(1+\mu\,\lambda^{-1}\right)\Big(\lambda\,(\lambda+\kappa)^{-1}(\theta\,(1-\alpha)+\phi\,\lambda) - (\lambda\,\phi+\omega\,(1-\alpha))\Big)\int\limits_0^\infty \mathrm{Exp}\left\{(\mu-\kappa)\,t\right\}\delta t\end{aligned} \tag{51}$$

Using Equation (48) above, the solution to Equation (51) above is...

$$I = R_0\left(1+\mu\,\lambda^{-1}\right)\Big(\lambda\,(\lambda+\kappa)^{-1}(\theta\,(1-\alpha)+\phi\,\lambda) - (\lambda\,\phi+\omega\,(1-\alpha))\Big)\left(\kappa-\mu\right)^{-1} \tag{52}$$

References

[1] Gary Schurman, *Modeling Technology Product Revenue,* December, 2019.

[2] Gary Schurman, *Capitalizing Research And Development Expenditures,* December, 2019.

[3] Gary Schurman, *Technology Company ROI And NPV,* December, 2019.

[4] Gary Schurman, *Recasting The Dividend Discount Model As A Return Model,* May, 2019.